On Our Understanding of Magnitudes

I recently read in the *Economist* a column by ''Johnson,'' the author of interesting, periodic essays on language. The latest essay, entitled *Beyond the giga-zone*, dealt with the language of prefixes for large and small numbers.

While a primary use of these prefixes is for mathematical and scientific terminology, the prefixes have indeed crept into everyday language. For example, Johnson cited several current uses of the prefix 'giga,' such as in 'Gigafactory,' to denote an exceptionally large factory. Another prefix currently in widespread use is 'nano,' such as in 'Nanotechnology,' referring to technology on exceedingly small scales.

In computing and science, these prefixes have very precise meaning. When I first became involved in statistical computing in the early 1960s, with a computer the size of a room, the memory of our first computer was 64K (kilobytes, thousand bytes).

After some years, technology had so advanced that a term for million bytes was needed. The prefix 'mega' was selected to denote one million. The word 'megabyte' stands for one million bytes of memory. The prefix 'mega' is derived from Greek, meaning 'great.' At the time of the original use of the term megabyte, it represented a huge amount of memory in the computer world.

Before too long, computer memory increased to hundreds of megabytes. The next prefix chosen was 'giga;' to represent 1,000 megabytes, in other words, one billion bytes of memory. The term 'giga' comes from the Greek, meaning 'giant.' At the time of its first usage, a gigabyte represented an immense amount of memory.

Not much later, a term was needed for a an even larger amount of memory. That led to the use of the prefix 'tera,' derived from the Greek, meaning 'monster.' The term 'terabyte' represents 1,000 gigabytes, or one million megabytes, or one trillion bytes.

Modern technology has also required prefixes for very small quantities, such as a billionth of a meter. Here, the prefix ‘nano’ is now being used. The term is derived from the Greek word for ‘dwarf.’ The prefix has also found its way into general usage, such as in ‘Nanotechnology,’ referring to technology dealing with very small objects. As technology advances, the next prefix needed will be Pico for a trillionth. This prefix is derived from the Spanish word for ‘a little bit.’

In scientific writing, use of prefixes to denote very large numbers would be much too cumbersome. Instead, in the decimal system, integer exponents to the power of 10 are used, where the exponent shows the number of zeros to the right of 1.

Thus, we have:

$10^0 = 1$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1{,}000$

$10^6 = 1{,}000{,}000$ (million)

$10^9 = 1{,}000{,}000{,}000$ (billion)

$10^{12} = 1{,}000{,}000{,}000{,}000$ (trillion)

A magnitude such as 1 million may have different significances for different persons. For example, $1 million may be a modest amount for a billionaire, but it may signify 30 year’s salary for a blue-collar worker.

For exceptionally large numbers, it is difficult, if almost impossible to comprehend them. For example, 10^{18} represents one billion billion. One billion is a large enough number to comprehend as it is. For example, if the federal government spent one dollar each *second*, it would take almost 32 *years* to spend $1 billion. To go beyond one billion to a billion billion for me is almost impossible to comprehend. The late Sen. Everett Dirksen is supposed to have said in the discussion of the federal budget dealing with billions of dollars: “A billion here, a billion there, and pretty soon you’re talking about real money.”

Sometimes, a comparison can be helpful. For example, our Milky Way, one of billions of galaxies, is itself an immense galaxy, containing billions of stars. Its observable mass is reported to be equal to 10^{12} solar masses. Our sun, one of billions of stars in the galaxy, is itself immense, containing almost 99.86% of the mass of the entire solar system.

Another example of the use of comparative size to get some comprehension of the size of the Milky Way is to consider the distance of Neptune from the sun. If the Solar System out to Neptune were the size of a US quarter, the Milky Way would be approximately the size of the contiguous United States.

But even then, we do not have a full comprehension of enormous quantities until we realize that the Milky Way, immense as it is, is but one of billions of galaxies in the universe.

Just as we have difficulty in comprehending large magnitudes, small magnitudes are also difficult to comprehend. In science, small quantities are expressed in the same way as large quantities, as powers of 10, but the exponent of 10 is negative which denotes the reciprocal.

Thus:

10^{-1} equals $1/10^1$ or 1/10 or 0.10

10^{-2} equals $1/10^2$ or 1/100 or 0.01

10^{-6} equals $1/10^6$ or 1/1,000,000 or 0.000001

The weight of a mosquito is 2.5 x 10^{-6} kilograms. Proceeding further down the scale, the weight of the *E. coli* bacterium is 1 x 10^{-15} kg, and the mass of the elementary particle electron is 9.1 x 10^{-31} kg.

Now it is just as difficult to comprehend the tiny magnitude 10^{-31} as it is to comprehend the enormously large quantity 10^{52}. Even quantities that are not that extreme are difficult to comprehend. For example, measurements of time to the 10^{-6} second, in other words, to the millionth of a second, are difficult for us to comprehend since even a whole second is a very short duration.

This introductory discussion turned out to be longer than I had anticipated, but I needed to be able to speak of a 10^{-31} world, a 10^{50} world and a 10^{0} world, the latter being the world familiar to us.

As I reflected on these magnitudes, I contemplated what if I were in the same world as an electron, namely a 10^{-31} world. From that perspective, our own 10^{0} world would be beyond any imagination, billions upon billions upon billions larger than the world of the electron. How could an electron, if it were sentient, imagine a universe in which the earth is but a speck of dust and in which a second is only flicker of time.

For some time, I have thought about our universe with questions such as: Is there life elsewhere in the universe? If so, can we communicate in any meaningful way, given that the speed of light (186,000 miles per second) is the fastest for any possible communication and that the nearest star is four light-years away? Communication across the Milky Way would take 100,000 years.

But while preparing this essay and contemplating the comprehension of large and small magnitudes, some additional questions have come to mind that go beyond our universe and reach out to our entire cosmos.

Are there other universes besides ours? Is the process of creating universes a continuous process in which new universes are continuously being created, like ours with the Big Bang? Do universes have a finite life or infinite life? Current science tells us that the expansion of our universe is accelerating, which would imply indefinite expansion. Will “dark matter,” still a mystery for us, or some other force be a brake on the expansion that will lead to a collapse of the universe, returning to the starting point of our universe as a singularity in a mathematical model, and then create another universe with a Big Bang?

Since I am in the last stage of my life, I will not be able to find out what science will learn about the cosmos beyond our universe. Perhaps I should follow the advice of Voltaire in *Candide*, where Pangloss, after many misadventures in life, concludes that it is best to return home and tend to one’s garden.

John Neter   May 8, 2022