School Paper Towels: Effective or a Waste of Money?

Introduction

Students across the country have noticed that their school-provided paper towels are all but effective. Cleaning up spills takes up an excessive amount of paper, and it seems clear that school-brand paper towels are ineffective and a waste of money. We wanted to add evidence to back our experiences, so we hoped to prove that our district-provided paper towels were underperforming other paper towels for its price point.

Statistical Question

Is there a positive linear relationship between price ($\$ / ft^2$) and the absorption rate ($grams\ of\ water / ft^2$) of paper towels? If so, does the brand our school district uses underperform other paper towels for its price point?

- H_0: $\beta = 0$
- H_a: $\beta > 0$
- Where β is the true slope of the population least squares regression line for the price ($\$ / ft^2$) vs. the absorption ($grams\ of\ water / ft^2$) of paper towels.

Data Collection: *Towel Brands Tested and Price Calculation (7 total)*

Towel Brand	Price/Pack	Rolls/Pack	Price/Roll	Ft^2/roll	Price/ft^2	Source
District	$84.89	6	$14.15	533.3333	**$0.02653**	MSC Direct
Scott Shop	$20.79	10	$2.08	39.4931	**$0.05264**	Costco
Bounty	$30.19	12	$2.52	54.7784	**$0.04593**	Costco
Kirkland	$25.19	12	$2.10	85.5556	**$0.02454**	Costco
Walgreens	$5.29	6	$0.88	39.4167	**$0.02237**	Walgreens
Viva	$61.46	24	$2.56	87.0833	**$0.02941**	CleanItSupply
Brawny	$59.89	16	$3.74	100.833	**$0.03712**	CleanItSupply

Location

We carried out the study for all paper towel brands in the afternoon on May 11, May 17, and May 22 in our school's physics classroom, which gave us access to highly-accurate scales and allowed us to keep external factors like temperature and humidity constant.

Selection: *We utilized three stages of random selection to minimize bias*

- **Selection Stage 1: Package Selection:** From the paper towel packages stocked at our local stores, we used a random number generator to randomly select one package.
- **Selection Stage 2: Roll Selection:** From a selected package, we used a random number generator to randomly select one roll
- **Selection Stage 3: Sheet Selection:** From a selected roll, we cut out 20 $4in\ x\ 4in$ paper towel squares. We randomly selected 5 using a random number generator.

Experiment

- **Preparation**
    1. Find two bowls with enough area to fully submerge a flat $4in\ x\ 4in$ paper towel.
    2. Fill one bowl with 250g of room temperature tap water.
        a. *This bowl will be used to soak the paper towel.*
    3. Place the other bowl onto a digital scale
        a. *This will hold the towel while its weight is being measured.*
- **Procedure**
    1. Place a dry $4in\ x\ 4in$ paper towel square into the bowl on the scale. Tare the scale. *Ensure the weight reads 0 grams before proceeding.*
    2. Place the paper towel square flat on the water (of the water bowl) for 20 seconds.
    3. Remove the paper towel square, holding two corners on the same side.

4. Hold the square above the water for 20 seconds to allow excess water to drip back into the bowl.
5. Place the wet paper towel square back into the bowl on the scale.
6. Record the weight (grams) displayed on the scale. *This weight reflects the amount of water absorbed by the paper towel.*
7. *Cleanup*: Discard the wet paper towel square. Dry the bowl on the scale using scrap paper towels. Refill the water bowl until it contains 250 grams of water again.

   Repeat all steps in the "Procedure" section for the remaining paper towel brands and their selected squares.

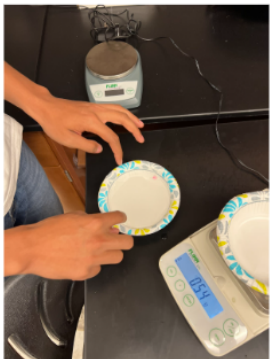
Notes

We collected data on all paper towel brands, including the paper towels provided by the district, on the days and locations mentioned above. However, we did not use the school brand's data when computing the LSR line. This way, we were able to compare the school towels' performance to that of the other brands. In addition, while we used $4in\ x\ 4in$ squares in the study, we multiplied the recorded absorption by three to determine the absorption for a square foot of paper towel. This scaling relies on the assumption that absorption and area have a positive linear relationship.

(Taring scale with paper towel)

(Soaking paper towel for 20 seconds)

(Letting towel drip for 20 seconds)



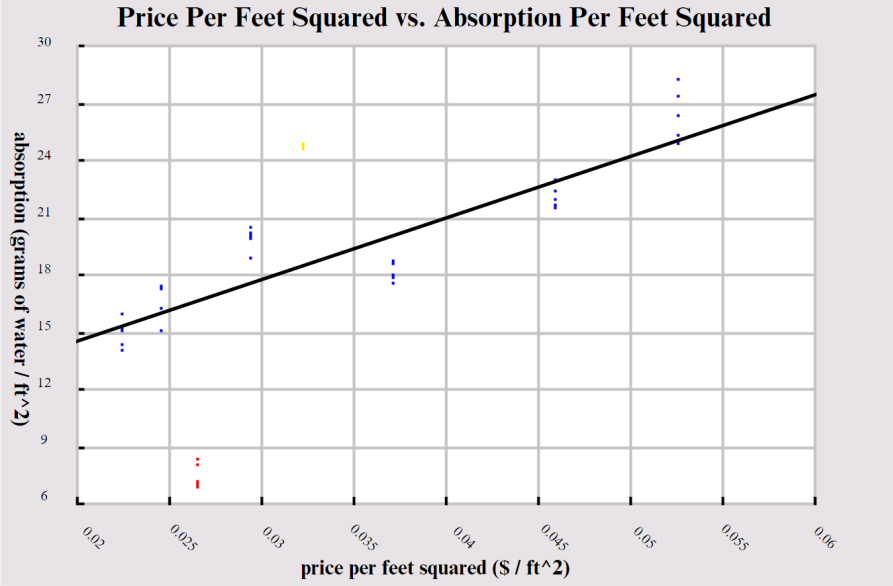
(Soaking paper towel for 20 seconds)

Data Display: Raw Data, Graphs, and Summary Statistics

Raw Data

Towel Brand	*Price/ft^2*	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
District	*0.0265281*	8.1	8.34	7.11	6.81	7.08
Scott Shop	*0.0526422*	25.29	28.29	26.34	27.42	24.84
Bounty	*0.0459274*	21.72	23.01	21.57	21.9	22.32
Walgreens	*0.0223679*	14.1	15.12	14.37	15.99	15.27
Kirkland	*0.0245357*	17.43	15.12	16.2	15.03	17.22
Viva	*0.0294067*	20.43	20.19	19.98	18.81	19.86
Brawny	*0.0371220*	17.49	18.69	17.85	17.94	18.57

Graph (Scatterplot with LSR Line)

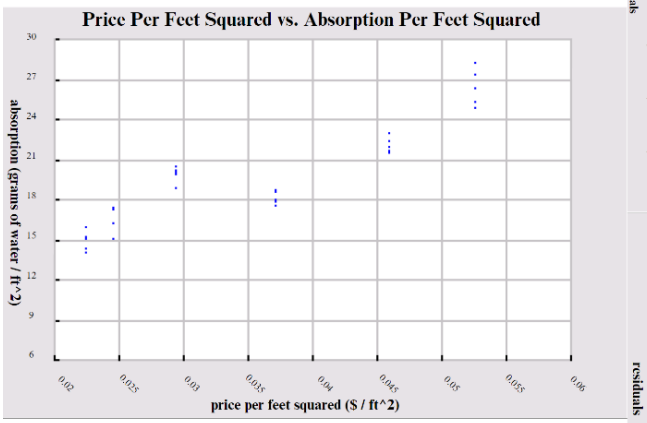


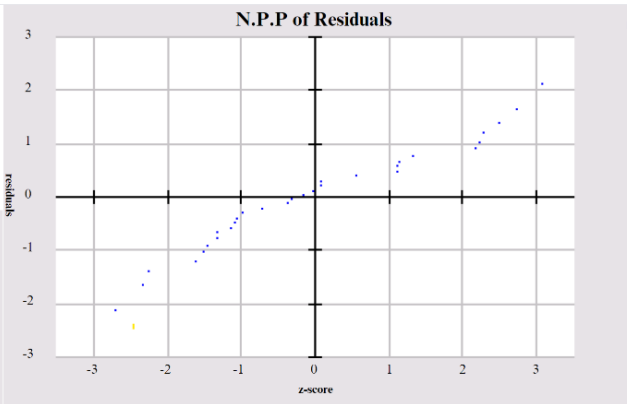
This graph shows that there is a strong positive linear relationship between price ($ / ft^2) and absorption ($grams\ of\ water\ /\ ft^2$) and that the district-provided paper towel brand (the red group of data around $0.027) underperforms at its price point. We will conduct a hypothesis test to prove this.

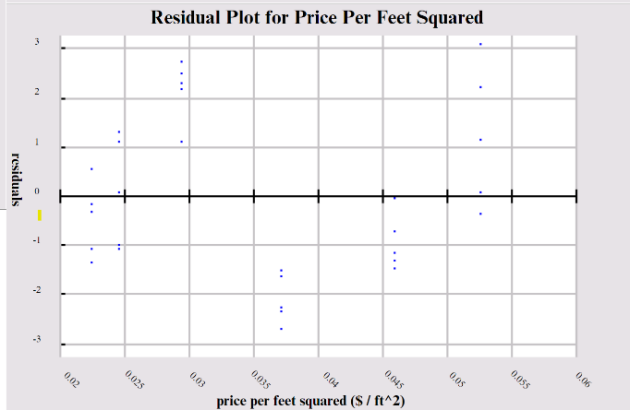
Summary Statistics

- **Least-Squares Regression Line**
  - $\hat{y} = 323.5309x + 8.1859$
  - $\hat{y}$: Predicted absorption per feet squared ($grams\ of\ water\ /\ ft^2$) for a paper towel
  - x: Price ($\$/ft^2$) of a paper towel
- r **(Correlation Coefficient)**
  - $r = 0.91207$
  - The correlation coefficient shows us the strength and direction of the linear relationship between price ($\$/ft^2$) and absorption per feet squared ($grams\ of\ water\ /\ ft^2$) in the data. Because the value is positive, we know the relationship in our data is also positive. The high value of 0.91207 also indicates that there is a strong linear relationship between the variables.
- r^2 **(Coefficient of Determination)**
  - $r^2 = 0.83188$
  - The coefficient of determination shows us that 83.188% of variation in the absorption per feet squared ($grams\ of\ water\ /\ ft^2$) is accounted for by the LSR line using price ($\$/ft^2$) as the explanatory variable. The relatively high value of 0.83188 gives us another indicator that our LSR model is a good fit because a high proportion of variation is accounted for by the LSR line.

- s **(Standard Deviation of the Residuals)**
    - $s = 1.6347$ grams
    - The standard deviation of the residuals shows us that on average, our LSR line is off by 1.6347 grams when predicting the absorption per feet squared ($grams\ of\ water\ /\ ft^2$) of paper towels using price ($\$/ft^2$). This value indicates that our model is a good fit because it is relatively small (less than 2 grams).
- b **(Slope)**
    - $b = 323.5309$
    - Our model predicts that for every 1 dollar increase in the price ($\$/ft^2$) of a paper towel, its absorption per feet squared ($grams\ of\ water\ /\ ft^2$) will increase by 323.5309 grams.
- **Outlier**
    - The school district brand is a definite outlier, primarily because of its large negative residual of 9.2812 grams (using the mean of the five trials). This residual indicates that the school district brand underperforms its predicted absorption by 9.2812 grams of absorption per square foot, showing that the school district isn't getting the best "bang for their buck" paper towels.







Data Analysis

Condition Checking: *We used the LINER condition check for our linear regression t-test*

Linear

The criterion for linearity is met. Visually, there is a clear linear relationship in the scatterplot between price (ft^2) and the absorption ($grams\ of\ water\ /\ ft^2$) of paper towels. The scatterplot has a strong positive linear pattern with no other clear pattern. In addition, the residual plot is approximately random with no clear pattern.

Independence: *10% rule must be met*

1. $N >= 10(total\ square\ footage\ used\ in\ the\ sample\ for\ a\ brand)$
2. $N >= 10(\frac{5}{3}ft^2)$
3. $Total\ Area\ of\ Each\ Roll >= 16.6667\ ft^2$

The criterion for independence has been met. In our price calculation spreadsheet, all the rolls we selected squares from had a total area greater than $16.6667\ ft^2$. In addition, we know the observations were independent from each other because we reset the amount of water in the container after each trial, so no one observation had an effect on another.

Normality

The criterion for Normality has been met. The Normal Probability Plot of the residuals appears to have a strong linear pattern, indicating that the data is approximately Normally distributed.

Equal Variance

The criterion for equal variance is met. The scatterplot of the residuals appears to have an approximately equal number of positive (13) and negative (17) values.

Random

The criterion for randomness is met. We had several stages of random selection.

Hypothesis Test: *Linear Regression t-test at* $\alpha = 0.05$

$b = 323.5309 \mid s = 1.6347 \mid n = 30 \mid k$ (degrees of freedom) $= 28 \mid s_x = 0.01105$

$$SE_b = \frac{s}{s_x\sqrt{n-1}} = \frac{1.6347}{0.01105\sqrt{30-1}} = 27.4711$$

$$t = \frac{b-\beta}{SE_b} = \frac{323.5309-0}{27.4711} = 11.7771$$

$$p = P(t > 11.7771) = 1.1577 * 10^{-12} \approx 0$$

Discission of Potential Errors

- **Type I Error:** A Type I Error would mean that we conclude there is a positive linear relationship between price and absorption, when there really isn't. This would mean that we would continue onwards to compare the district's paper towels to the (incorrect) LSR line, potentially leading us to give the district an incorrect recommendation.
- **Type II Error:** A Type II Error would mean that we do not conclude there is a positive linear relationship between price and absorption, when there really is one. This would mean we would not proceed to compare the district's paper towels to the (correct) LSR line, leading us to make no recommendation to the district even though we could have.

We chose a significance level of 0.05 to serve as a "middle ground" between a Type 1 and Type 2 error because we didn't feel either error was "worse" than another.

Conclusion

Since our p-value of $1.1577 * 10^{-12}$ is less than $\alpha = 0.05$ we reject H_0 and have sufficient evidence to conclude there is a positive linear relationship between price ($\$ / ft^2$) and the absorption ($grams\ of\ water / ft^2$) of paper towels similar to those in our study. Because the residual for the district-provided paper towels is so large, it can be concluded that our district is not purchasing the paper towel brands it should be for the given price point. With this, we can recommend the district purchase higher-quality paper towels for a comparable price point.

Reflection

We acknowledge that there may have been several flaws in our study. We chose to use $4in\ x\ 4in$ squares of paper towels and multiplied absorption by a factor of three to estimate the absorption of a square foot of paper towel to save time and money, but collecting data on a full square foot of paper towel may have reduced variation in our data.

In addition, we acknowledge that there was variation caused by inconsistent cutting methods. We cut our paper towels using classroom scissors and rulers, which resulted in our paper towel squares having weights that varied by up to 0.05 grams within the same brand. To

somewhat offset the variation caused by this, we re-tared the scale before each measurement. However, finding a way to accurately and consistently cut the paper towels definitely would have helped reduce variation in the data.

We would also like to consider alternate methods to evaluate the effectiveness of paper towel brands. When evaluating whether the district-provided paper towels were suitable for the price point, we only measured the absorbency. In reality, there can be many factors that influence the effectiveness of a paper towel that go beyond absorbency. Objective factors, like thickness or material (i.e. wood pulp, bamboo, recycled fibers, etc.), would be interesting to measure. In addition, subjective factors, such as personal ratings, would be equally important to measure. Subjective data could be collected by randomly assigning paper towel brands to participants and asking them to rate the quality on a scale of 1-10. This would add depth to the study and strengthen the results.

Overall, we can conclude that the district-provided paper towels were just as bad as we expected them to be. Hopefully, with these findings, we can convince the district to purchase higher-quality paper towels for a similar price point.